Math 308O, Midterm 2 Name: _____

Signature: _____

Student ID #: _____ Section #: _____

- You are allowed a Ti-30x IIS Calculator and one 8.5×11 inch paper with handwritten notes on both sides. Other calculators, electronic devices (e.g. cell phones, laptops, etc.), notes, and books are **not** allowed.
- *All* answers on the exam must be justified. You will receive at most 1 point out of 10 for an answer without any explanation.
- Place a box around your answer to each question.
- Raise your hand if you have a question.

1a	/10
1b	/10
2a	/10
2b	/10
3	/10
4	/10
T	/60

Good Luck!

(1) [10pts] Let A and B be the following equivalent 4×6 matrices.

$$A := \begin{bmatrix} 0 & 0 & -1 & 3 & 0 & 1 \\ 1 & 2 & 1 & 1 & 0 & 0 \\ 1 & 2 & 2 & -2 & 0 & -1 \\ 0 & 0 & -3 & 9 & 0 & 3 \end{bmatrix}, \quad B := \begin{bmatrix} 1 & 2 & 0 & 4 & 0 & 1 \\ 0 & 0 & 1 & -3 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) [10pts] Find a basis for $\text{Null}(A)$ and determine the nullity of A.

(b) [10pts] Determine if the vector $\mathbf{b} = \begin{bmatrix} -1 \\ -2 \\ 0 \\ 0 \end{bmatrix} \in \text{Col}(A)$. (It may help to find a basis of $\text{Col}(A)$ first.)

(2) Let $A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & -1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$.

(a) [10pts] Compute A^3.

(b) [10pts] Determine if A is invertible and, if so, compute A^{-1}.

(3) [10pts] Give an example of a linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ with the following two properties:
(a) T is *not* one-to-one and
(b)

$$\text{range}(T) = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 : x - y + 2z = 0 \right\};$$

OR explain why this is not possible. If you give an example, you must include an explanation of why your linear transformation has the desired properties.

(4) [10pts] Give an example of a 3×3 non-diagonal matrix A with $\det(A) = -1$ and $A^2 = I$, OR explain why this is not possible. If you give an example, you must include an explanation of why your matrix has the desired properties. (*Hint:* It may help to think geometrically!)